B.Sc. Part I (Hons.) 2nd Paper

Integral Calculus

Reduction formula (contd.)

I. Find the reduction formula for $\int \sin^n x\, dx$.

Soln Let $I_n = \int \sin^n x\, dx$ ——— (1)

$\Rightarrow I_n = \int \sin x . \sin^{n-1} x\, dx$. Now, integrating by parts, we get

$$\Rightarrow I_n = \sin^{n-1} x \int \sin x\, dx - \int \left[\frac{d}{dx}\{\sin^{n-1} x\} \int \sin x\, dx \right] dx$$

$$\Rightarrow I_n = \sin^{n-1} x . (-)\cos x - \int (n-1) \sin^{n-2} x . \cos x . (-)\cos x\, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x . \cos^2 x\, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x . (1 - \sin^2 x)\, dx$$

$$\Rightarrow I_n = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x\, dx - (n-1) \int \sin^{n-2} x . \sin^2 x\, dx$$

$$\Rightarrow I_n = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x\, dx - (n-1) \int \sin^n x\, dx \quad \text{——— (2)}$$

From (1), $I_n = \int \sin^n x \, dx$

Replacing n by $n-2$, we get

$$I_{n-2} = \int \sin^{n-2} x \, dx$$

Putting this value and using (1) in eq(2),
we get

$$I_n = -\cos x . \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$\Rightarrow I_n + (n-1) I_n = -\cos x . \sin^{n-1} x + (n-1) I_{n-2}$$

$$\Rightarrow I_n (1 + n-1) = -\cos x . \sin^{n-1} x + (n-1) I_{n-2}$$

$$\Rightarrow n I_n = -\cos x . \sin^{n-1} x + (n-1) I_{n-2}$$

$$\Rightarrow I_n = -\frac{1}{n} \cos x . \sin^{n-1} x + \frac{n-1}{n} I_{n-2}$$